Light thresholds in
Grand Unified Theories

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Abstract

In a generic Grand Unified Theory with a relatively small dispersion of the spectrum around the Z-boson and the unification masses, a connection is established, exact at one loop level, between M_Z, G_{F}, $\alpha(M_Z)$ and the strong coupling constant $\alpha_3(M_Z)$. At this level of precision, this avoids the logical and phenomenological inconsistency of predicting $\alpha_3(M_Z)$ by means of the electroweak couplings as extracted from the data in the Standard Model rather than in the complete theory. Attention is paid to the independence of the physical results from regularization and/or renormalization schemes.

As a particularly relevant example, the analysis is specialized to the case of the Minimal Supersymmetric Standard Model, with emphasis on light charginos and neutralinos.

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1 Introduction

The observed unification of the strong and electroweak coupling constants in a supersymmetric Grand Unified Theory is among the very few direct experimental results (the only one at present?) with a clear interpretation in terms of a non-standard theory of the strong and the electroweak interactions [1]. Needless to say, such an interpretation requires a number of strong theoretical hypotheses about the theory in its super-high energy regime. These hypotheses, however, can be formulated in a clear way. We would summarize them as follows:

i) the dispersion of the spectrum of the theory around the unification scale $M_{\rm G}$ does not distort the evolution of the couplings in a significant way;

ii) equally negligible are the effects related to the proximity of $M_{\rm G}$ to the Planck scale $M_{\rm Pl}$, as possibly due to the presence of higher dimensional operators, scaled by inverse powers of $M_{\rm Pl}$.

Once these hypotheses are made, the possibility to determine the effects of the unification of the couplings in the low energy theory is only limited by the level of knowledge of the low energy theory itself and can in principle be pushed to an arbitrary level of accuracy.

Along these lines, a consistent treatment of all possible effects up to two loops in the β-function for the running of the couplings and including one loop "threshold corrections" is given in the relevant literature [2]. Recently, even one loop non-logarithmic corrections have been considered [3, 4], that scale like powers of $M_Z/m_{\rm S}$, the ratio between the Z-mass and the mass of any particle present in the low-energy spectrum. As long as one expects some supersymmetric particle with a mass comparable to M_Z, this is both logically necessary and, maybe, phenomenologically relevant. The numerical results of ref. [3] indicate that this can be the case.

A common feature of all these works, however, is the assumption of an initial condition for two of the three relevant couplings at M_Z, obtained from a fit of the various experimental data *inside* the Standard Model. More explicitly, to predict the strong coupling constant $\alpha_3(M_Z)$, one takes a value for the electroweak couplings $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ or, alternatively, for $\alpha(M_Z)$ and $\sin^2\theta_{\rm W}(M_Z)$ using a Standard Model fit of the electroweak precision data. Clearly, in so far as $M_Z \approx m_{\rm S}$, one makes in this way an error of the same order of the effects that are being included by a proper treatment of the thresholds corrections in the evolution of the couplings. Here we would like to remedy to this shortcoming of the present analysis, by replacing the inputs for $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ with direct experimental quantities.

In principle, thinking of all the different electroweak precision observables, this would seem to require a complicated fitting procedure involving all of them. In practice, this is not the case. A unified theory, with a given dispersion of the spectrum around M_Z and $M_{\rm G}$, can be viewed as dependent on three parameters (suitably defined): the grand scale, the low energy scale and the unified coupling. As such, three measurements ("basic observables") are required to fix the theory and a fourth one (at least) to test it. As is well known, for reasons of experimental precision and theoretical cleanliness, the three basic observables that emerge are: M_Z, the Fermi constant $G_{\rm F}$, as measured in μ decay, and the electromagnetic fine structure constant at the Z mass $\alpha(M_Z)$. On the other hand, any observable with a significant dependence on the strong coupling can serve as a test of the unified theory. Since none, in this case, clearly dominates as yet over the others, we shall take $\alpha_3(M_Z)$ itself, defined in the $\overline{\rm MS}$ scheme, as commonly done in the literature. We will therefore establish a connection between M_Z, $G_{\rm F}$, $\alpha(M_Z)$ and $\alpha_3(M_Z)$ as function of the particle spectrum, taking into account all one loop effects in an exact way. Needless to say, the particle spectrum will have to be consistent with all data so far, including the electroweak precision tests. In the case of the supersymmetric particles, once the constraints from production experiments are satisfied, the virtual effects are sufficiently small to be generally consistent with observations [5]. This is certainly the case if the stop-sbottom splitting is not too large relative to the mean mass and a tiny region of the parameter space where charginos have a mass of $45 \div 50$ GeV is perhaps also avoided. Notice, on the other hand, that this is not an a priori prejudice to the possible relevance of the considerations developed in this work, due to the different quality of the determinations of M_Z, $G_{\rm F}$ and $\alpha(M_Z)$ relative to all other electroweak precision observables.

2 General reference formulae

We aim at establishing in this section the general connection between M_Z, G_F, $\alpha(M_Z)$ and $\alpha_3(M_Z)$. Following Weinberg [6], the program for calculating $\alpha_3(M_Z)$ to the required accuracy proceeds in three steps, as follows:

a) Determine the renormalized $SU(3) \otimes SU(2) \otimes U(1)$ couplings $\alpha_i(\mu_H)$, $i = 1, 2, 3$ at a scale μ_H (H = heavy) such that $\mu_H^2/M_G^2 \ll 1$, but still $\ln(\mu_H/M_G) = \mathcal{O}(1)$. Denoting by $\delta\alpha_i(\mu_H)$ the one loop contributions from the heavy particles to the vacuum polarizations of the light gauge fields, we write, in terms of the unified bare coupling α_G

$$\frac{1}{\alpha_i(\mu_H)} = \frac{1}{\alpha_G} - \frac{\delta\alpha_i(\mu_H)}{\alpha_i^2}. \tag{1}$$

b) By means of the two loop β-function for the $SU(3) \otimes SU(2) \otimes U(1)$ gauge group, run the couplings down to a scale μ_L (L = light) such that $m_S^2/\mu_L^2 \ll 1$, but $\ln(m_S/\mu_L) = \mathcal{O}(1)$, where m_S is a typical light particle mass. In this running m_S can be neglected. One can write

$$\frac{1}{\alpha_i(\mu_L)} = \frac{1}{\alpha_i(\mu_H)} + b_i \ln \frac{\mu_H}{\mu_L} + \delta_i^{HL} \tag{2}$$

where b_i are the one loop β-function coefficients and δ_i^{HL} are the two loop terms with μ_H and μ_L replaced, in a consistent approximation, with M_G and M_Z respectively. One has

$$\delta_i^{HL} = \frac{1}{4\pi} \left[\frac{b_{it}^{(2)}}{6} \ln\left(1 + 10.5\,\lambda_t^2(M_G)\right) + \sum_{j=1}^{3} \frac{b_{ij}^{(2)}}{b_j} \ln \frac{\alpha_j(M_G)}{\alpha_j(M_Z)} \right], \tag{3}$$

where $b_{ij}^{(2)}$ are the two loop β-function coefficients and $\lambda_t(M_G)$ is the top Yukawa coupling at the unification scale.

c) From the inputs $\alpha_i(\mu_L)$, calculate the (suitably defined) $\alpha_i(M_Z)$, taking into account all one loop effects of the light particles in $\delta\alpha_i(M_Z)$:

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(\mu_L)} - \frac{\delta\alpha_i(M_Z)}{\alpha_i^2}. \tag{4}$$

From equations (1), (2), (4), it is a simple matter to obtain, by eliminating α_G and $\ln(\mu_H/\mu_L)$,

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{b_{12}}\left(\frac{b_{13}}{\alpha_2(M_Z)} - \frac{b_{23}}{\alpha_1(M_Z)}\right) + \delta_H + \delta_{HL} + \delta_L \tag{5}$$

where $b_{ij} \equiv b_i - b_j$ and

$$\delta_{HL} = \frac{1}{b_{12}}(b_{23}\delta_1^{HL} + b_{31}\delta_2^{HL} + b_{12}\delta_3^{HL}), \tag{6a}$$

$$\delta_H = -\frac{1}{b_{12}}\left[b_{23}\frac{\delta\alpha_1(\mu_H)}{\alpha_1^2} + b_{31}\frac{\delta\alpha_2(\mu_H)}{\alpha_3^2} + b_{12}\frac{\delta\alpha_3(\mu_H)}{\alpha_3^2}\right], \tag{6b}$$

$$\delta_L = -\frac{1}{b_{12}}\left[b_{23}\frac{\delta\alpha_1(M_Z)}{\alpha_1^2} + b_{31}\frac{\delta\alpha_2(M_Z)}{\alpha_3^2} + b_{12}\frac{\delta\alpha_3(M_Z)}{\alpha_3^2}\right]. \tag{6c}$$

Equation (5) is not yet what we want, however. To express $\alpha_3(M_Z)$ in terms of M_Z, G_F and $\alpha(M_Z)$, we need to connect them to $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$. Defining, as usual in the literature, an auxiliary mixing angle parameter through

$$s^2 \equiv \frac{1}{2}\left[1 - \sqrt{1 - \frac{4\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2}}\right], \qquad c^2 \equiv 1 - s^2, \tag{7}$$

such connection, exact at one loop, is given by

$$\alpha_1(M_Z) = \frac{\alpha}{c^2}\left[1 - \frac{\delta\alpha}{\alpha} + \frac{\delta c^2}{c^2} + \frac{\delta\alpha_1(M_Z)}{\alpha_1}\right]\frac{5}{3} \tag{8a}$$

$$\alpha_2(M_Z) = \frac{\alpha}{s^2}\left[1 - \frac{\delta\alpha}{\alpha} + \frac{\delta s^2}{s^2} + \frac{\delta\alpha_2(M_Z)}{\alpha_2}\right], \tag{8b}$$

where

$$\delta s^2 = -\delta c^2 = \frac{s^2c^2}{c^2 - s^2}\left[\frac{\delta\alpha}{\alpha} - \frac{\delta G_{\rm F}}{G_{\rm F}} - \frac{\delta M_Z^2}{M_Z^2}\right]$$

and $\delta\alpha$, $\delta G_{\rm F}$, δM_Z^2 are the full one loop corrections from all the light particles to $\alpha(M_Z)$, $G_{\rm F}$ and M_Z^2 respectively. This allows us to write down the result for $\alpha_3(M_Z)$ in the desired form

$$\frac{1}{\alpha_3(M_Z)} = \frac{b_{13}s^2 - \frac{3}{5}b_{23}c^2}{b_{12}\alpha(M_Z)} + \delta_{\rm H} + \delta_{\rm HL} + \Delta_{\rm L}, \tag{9}$$

where, in this case, the light particle contribution to the one loop correction $\delta_{\rm L}$ is replaced by

$$\Delta_{\rm L} = -\frac{1}{b_{12}}\left[b_{23}\frac{\Delta\alpha_1(M_Z)}{\alpha_1^2} + b_{31}\frac{\Delta\alpha_2(M_Z)}{\alpha_3^2} + b_{12}\frac{\delta\alpha_3(M_Z)}{\alpha_3^2}\right] \tag{10}$$

$$\frac{\Delta\alpha_1(M_Z)}{\alpha_1} = \frac{\delta\alpha}{\alpha} - \frac{\delta c^2}{c^2}, \tag{11a}$$

$$\frac{\Delta\alpha_2(M_Z)}{\alpha_2} = \frac{\delta\alpha}{\alpha} - \frac{\delta s^2}{s^2}. \tag{11b}$$

Finally, $\Delta\alpha_1(M_Z)$ and $\Delta\alpha_2(M_Z)$ are easily expressed in terms of commonly defined amplitudes [7]. One has

$$\frac{\Delta\alpha_1(M_Z)}{\alpha_1} = -F_{00}(M_Z^2) + \frac{1}{c^2 - s^2}\left[s^2(e_1 - \frac{\delta G_{\rm VB}}{G_{\rm F}}) - (2s^2e_3 + 2\frac{s}{c}\frac{A_{\gamma Z}}{M_Z^2}) - c^2e_4\right] \tag{12a}$$

$$\frac{\Delta\alpha_2(M_Z)}{\alpha_2} = -F_{33}(M_Z^2) - \frac{1}{c^2 - s^2}\left[c^2(e_1 - \frac{\delta G_{\rm VB}}{G_{\rm F}}) - (2s^2e_3 + 2\frac{s}{c}\frac{A_{\gamma Z}}{M_Z^2}) - s^2e_4\right] \tag{12b}$$

where, from the vacuum polarization amplitudes ($i,j = W, Z, \gamma$ and $i,j = 0, 3$ for B, W_3)

$$\Pi_{\mu\nu}^{ij}(q^2) = -ig_{\mu\nu}[A_{ij} + q^2F_{ij}(q^2)] + q_\mu q_\nu \text{ terms,}$$

$$e_1 \equiv \frac{A_{33} - A_{WW}}{M_W^2} = \frac{A_{ZZ}}{M_Z^2} - \frac{A_{WW}}{M_W^2} + 2\frac{s}{c}\frac{A_{\gamma Z}}{M_Z^2}$$

$$e_3 \equiv \frac{c}{s}F_{03}(M_Z^2)$$

$$e_4 \equiv F_{\gamma\gamma}(0) - F_{\gamma\gamma}(M_Z^2)$$

and $\delta G_{\rm VB}$ is the one loop correction, except for vacuum polarizations (vertices, boxes, and fermion self-energies), to the μ-decay amplitude [7]. Notice that e_4 contains all the one loop contributions to the photon vacuum polarization which are not included in $\alpha(M_Z)$. Conventionally, we take $\alpha(M_Z)$ to contain only the corrections from all leptons and quarks except the top.

Before going to numerical results, we find it useful to make clear to what extent the final result, eq. (9), is independent from regularization and/or renormalization schemes. If, in place of $\alpha_3(M_Z)$, we had used a direct physical observable, like, e. g., the hadronic width of the Z-boson, no reference to any regularization or renormalization scheme should have been made at all, except, possibly, in the intermediate steps of the calculation.

The first term in the right-hand side of eq. (9), dependent on physical observables only, has an absolute meaning. The heavy particle correction term $\delta_{\rm H}$ has also an intrinsic definition, being obtained through the gauge invariant [6] one loop contributions to the vacuum polarizations of the light gauge bosons. Since $(\mu_{\rm H}/M_{\rm G})^2 \ll 1$, $\delta_{\rm H}$ does not depend on $\mu_{\rm H}$ and, because of unification, it is ultraviolet finite. In spite of that, it does however depend upon the regularization scheme. In terms of the masses $\{M_h\}$ of the heavy particles and their contributions b_i^h to the β-functions coefficients b_i for the three gauge group factors in $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$, one has

$$\delta_{\rm H} = \frac{1}{2\pi b_{12}} \sum_h (b_{23} b_1^h + b_{31} b_2^h + b_{12} b_3^h) \ln \frac{M}{M_h} + \frac{2b_{13} - 3b_{12}}{12\pi b_{12}}, \tag{13}$$

with the mass independent term given in the usual $\overline{\rm MS}$ scheme. Note also that $\delta_{\rm H}$ does not depend on the overall scale M since, due to unification,

$$\sum_h b_i^h = b_{\rm G} - b_i. \tag{14}$$

where $b_{\rm G}$ is the one loop coefficient of the β-function for the unified group.

Unlike $\delta_{\rm H}$, the light particle correction term $\Delta_{\rm L}$ does not have an intrinsic definition, but depends on the definition of $\alpha_3(M_Z)$ via $\delta\alpha_3(M_Z)$. We stick to the usual $\overline{\rm MS}$ definition of $\alpha_3(M_Z)$, so that

$$\frac{\delta\alpha_3(M_Z)}{\alpha_3^2} = -\frac{b_3}{4\pi}\left(\frac{2}{d-4} - \gamma_{\rm E} + \ln 4\pi + \ln \frac{M_Z^2}{\mu_{\rm MS}^2}\right) + \sum_\ell \frac{b_3^\ell}{4\pi} \ln \frac{M_\ell^2}{M_Z^2}, \tag{15}$$

where d is the dimension of space time, $\mu_{\rm MS}$ is the dimensional regularization mass scale, and the sum extends over all the light particles heavier than M_Z. The calculation of $\Delta_{\rm L}$ is completed by the expressions for $\Delta\alpha_1(M_Z)$ and $\Delta\alpha_2(M_Z)$, which are intrinsically defined in any given theory. The overall $\Delta_{\rm L}$, although ultraviolet finite, has however a regularization dependence which would compensate the regularization dependence of $\delta_{\rm H}$, if we had computed the strong interaction corrections to a physical observable instead of $\alpha_3(M_Z)$ itself. The compensation comes about as follows. For example in dimensional regularization, different extensions of the Lorentz and Dirac indices (algebra of γ matrices in d dimensions, definition of γ_5, etc.) amounts to obtain a finite contribution to the various $\delta\alpha_i$ of the form

$$\frac{\delta_{\rm reg}\alpha_i}{\alpha_i^2} = b_{i\rm V} f_{\rm V} + b_{i\rm F} f_{\rm F} + b_{i\rm S} f_{\rm S} \tag{16}$$

where $b_{i\rm V}, b_{i\rm F}, b_{i\rm S}$ are the contributions to the β-function coefficients of the Vector, the Fermions and the Scalars respectively, whereas $f_{\rm V}, f_{\rm F}$ and $f_{\rm S}$ are constants dependent on the specific regularization scheme [2, 6, 8]. If eq. (16) is inserted in all $\delta\alpha_i$, or $\Delta\alpha_i$, both in $\delta_{\rm H}$ and $\Delta_{\rm L}$, a full cancellation takes place for any $f_{\rm V}, f_{\rm F}, f_{\rm S}$ since the vectors, the fermions and the scalars, light and heavy, form individual complete multiplets of the unified group. The cancellation is incomplete, because $\delta\alpha_3(M_Z)$ is defined in (15) without the extra regularization dependent term in (16), and is only recovered when the relation between $\alpha_3(M_Z)$ and the physical observable is established.

3   The case of the Minimal Supersymmetric Standard Model

We can now specialize the result of eq. (9) to the case of the Minimal Supersymmetric Standard Model [9]. The dominant term in the right hand side of eq. (9), using

$$
\begin{align}
M_Z &= 91.188 \pm 0.0044\,\text{GeV} \tag{17a}\\
G_{\rm F} &= 1.16637(2) \cdot 10^{-5}\,\text{GeV}^{-2} \tag{17b}\\
\alpha(M_Z) &= (128.87 \pm 0.12)^{-1} \tag{17c}\\
(s^2 &= 0.2312 \pm 0.0003) \tag{17d}
\end{align}
$$

and

$$b_1 = \frac{33}{5}, \qquad b_2 = 1, \qquad b_3 = -3,$$

m_t in GeV	$\Delta_{\mathrm{L}}^t(m_t)$	m_h in GeV	$\Delta_{\mathrm{L}}^h(m_h)$
120	-0.898	50	1.117
130	-0.977	60	1.140
140	-1.061	70	1.160
150	-1.151	80	1.177
160	-1.246	90	1.193
170	-1.347	100	1.207
180	-1.452	110	1.220
190	-1.564	120	1.232
200	-1.680	130	1.243
210	-1.803	140	1.254
220	-1.903	150	1.263

Table 1: numerical values of $\Delta_{\mathrm{L}}^t(m_t)$ and $\Delta_{\mathrm{L}}^h(m_h)$ for representative values of m_t and m_h.

gives

$$\alpha_3(M_Z)(\text{leading logs}) = 0.1155 \pm 0.0010. \tag{18}$$

For the two loop term δ_{HL}, also including a correction involving the top Yukawa coupling, one has [2]

$$\delta_{\mathrm{HL}} = -0.80 \pm 0.10. \tag{19}$$

Assuming no dispersion of the heavy particle spectrum, the heavy particle contribution (13) gives a negligible effect ($\delta_{\mathrm{H}} \approx 0.01$), so that, before the inclusion of Δ_{L}, $\alpha_3 = 0.1273 \pm 0.0020$.

Coming to the light particle correction term Δ_{L}, let us first consider the case where all the extra particles introduced by supersymmetry are heavy with respect to M_Z, so that all power law corrections in M_Z/m_{S} can be safely neglected. In this situation, apart from a contribution to the cancellation of the divergences, the supersymmetric particles only contribute to Δ_{L} with logarithmic terms, $\ln(m_{\mathrm{S}}/M_Z)$, that have been studied in the literature [2, 10]. On the other hand, also the various contributions to e_1, e_3, e_4, $F_{00}(M_Z)$, $F_{33}(M_Z)$ and $\delta G_{\mathrm{VB}}/G_{\mathrm{F}}$ from W, Z, γ, light fermions, top and standard Higgs boson, can be reconstructed from the literature [11]. By putting everything together, taking all supersymmetric particles degenerate at m_{S}, we find, for $m_t > M_Z/2$,

$$\Delta_{\mathrm{L}} = \Delta_{\mathrm{L}}^t(m_t) + \Delta_{\mathrm{L}}^h(m_h) + \Delta_{\mathrm{L}}^{\mathrm{susy}} \tag{20a}$$

$$\Delta_{\mathrm{L}}^t = \frac{45}{112\pi}\frac{1}{c^2-s^2}\left\{ \frac{4}{81}(16s^4 + 128s^2 - 73) - t - \frac{2}{9}\ln t - \right. \tag{20b}$$

$$\left. + \frac{4}{27}[(24s^2 - 9 - 32s^4) + (9 + 48s^2 - 64s^4)t]\left[1 - \sqrt{4t-1}\arcsin\frac{1}{\sqrt{4t}}\right]\right\}$$

$$\Delta_{\mathrm{L}}^h = 1.197 + \frac{45}{112\pi}\frac{1}{c^2-s^2}\left\{ \frac{h^2 s^2 \ln h}{(1-h)(h-c^2)} + \frac{hc^2 \ln c^2}{h-c^2} + \frac{h}{18} + \right. \tag{20c}$$

$$\left. + \frac{1}{9}(12 - 4h + h^2)\left[1 + (\frac{h}{h-1} - \frac{h}{2})\ln h - h\sqrt{\frac{4}{h} - 1}\arctan\sqrt{\frac{4}{h} - 1}\right]\right\}$$

$$\Delta_{\mathrm{L}}^{\mathrm{susy}} = \frac{19}{28\pi}\ln\frac{m_{\mathrm{S}}}{M_Z} \qquad \text{for } m_{\mathrm{S}}^2 \gg M_Z^2, \tag{20d}$$

where $t \equiv m_t^2/M_Z^2$ and $h \equiv m_h^2/M_Z^2$. The numerical values of Δ_{L}^t and Δ_{L}^h for representative values of m_t and m_h are given in table 1. As overall result, the prediction for $\alpha_3(M_Z)$ as function of m_t and m_h, taking $\delta_{\mathrm{H}} = 0$, $\delta_{\mathrm{HL}} = -0.80$ and, for sake of illustration, $\Delta_{\mathrm{L}}^{\mathrm{susy}} = 0$ is shown in fig. 1.

Of course our interest is in possibly significant deviations from these predictions for $\alpha_3(M_Z)$ coming from light supersymmetric particles like, e.g., gaugino-higgsinos or sfermions. On the contrary, for the purpose of the present discussion, we can safely neglect possible deviations in the Higgs system from
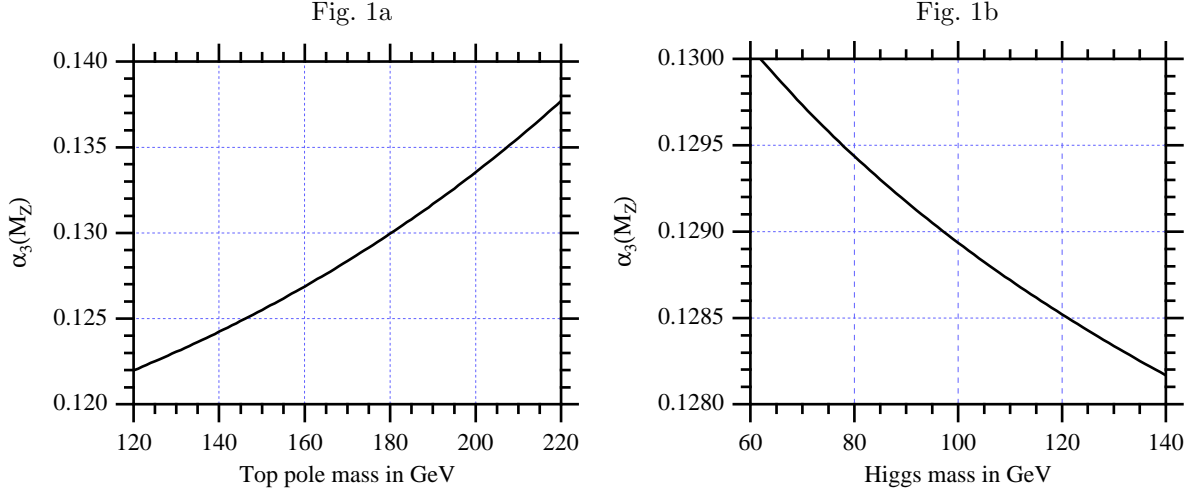
Figure 1: $\alpha_3(M_Z)$ as function of the top pole mass M_t for $m_h = M_Z$ (1a) and of m_h for $M_t = 175$ GeV (1b), taking $\Delta_{\mathrm{L}}^{\mathrm{susy}} = 0$ and $\delta_{\mathrm{HL}} = -0.80$.

the case of the Standard Model with a light Higgs. Let us consider a particular set of light super-symmetric particles. The corresponding contribution to Δ_{L}, exact to one loop, can be obtained from eq.s (10,12a,12b,15). We find

$$\Delta_{\mathrm{L}}^{\mathrm{susy}}(\text{light sparticles}) \quad = \quad 42.46(F_{00}^{\ell} - \frac{\alpha_1}{4\pi}b_1^{\ell}\Delta_Z) - 51.07(F_{33}^{\ell} - \frac{\alpha_2}{4\pi}b_2^{\ell}\Delta_Z) + \tag{21}$$
$$-91.29(e_1^{\ell} + \frac{\delta G_{\mathrm{VB}}^{\ell}}{G_{\mathrm{F}}}) + 80.44e_3^{\ell} + 82.68e_4^{\ell} - \sum_{\ell}\frac{b_3^{\ell}}{4\pi}\ln\frac{M_{\ell}^2}{M_Z^2},$$

where the sum extends over all the light sparticles heavier than M_Z,

$$\Delta_Z \equiv \frac{2}{d-4} - \gamma_{\mathrm{E}} + \ln 4\pi + \ln\frac{M_Z^2}{\mu_{\overline{\mathrm{MS}}}^2},$$

b_i^{ℓ} is the contribution to the β-function coefficients of the light supersymmetric particles and we denote by F_{00}^{ℓ}, F_{33}^{ℓ}, $\delta G_{\mathrm{VB}}^{\ell}$, e_1^{ℓ}, e_3^{ℓ}, e_4^{ℓ} their contributions to the corresponding functions and by M_{ℓ} their masses. The divergent pieces in $\Delta_{\mathrm{L}}^{\mathrm{susy}}$ have also been properly subtracted away.

The value of $\Delta_{\mathrm{L}}^{\mathrm{susy}}$ can be analyzed in the parameter space of the Minimal Supersymmetric Standard Model. We find that it is most likely to be relevant in the case of light charginos and neutralinos, illustrated in figures 2,3. In figure 2 we give in the usual parameter space [9] the contribution to Δ_{L} from charginos and neutralinos, for different values of $\tan\beta$. In the logarithmic scale that we use, the deviation from the purely logarithmic approximation distorts the contour plot from a pure straight and equidistant line pattern. Such distortion is significant for values of μ and M_2 (as renormalized at low energy) reasonably close to the boundary of the excluded region, defined by requiring the lightest chargino to be heavier than 45 GeV. Numerically the deviation from the logarithmic approximation varies from -1.0 to 0.15 for $\tan\beta = 1.5$ and from -0.65 to 0.0 for $\tan\beta = 40$. The relative size of this effect, from point to point in parameter space, can be comparable to the two loop β-function correction δ_{HL}.

Always with the focus on possibly light charginos and neutralinos, we have collected in figures 3 all the various contribution to $\alpha_3(M_Z)$. There we take $m_t = 175$ GeV, $m_h = M_Z$, the gluino mass as consistently determined from M_2 and the usual unification condition, and, for all other heavy particles, sfermions and heavy Higgs bosons, $\Delta_{\mathrm{L}} = 1$. Within the assumption stated in the introduction, and for fixed values of the parameters as explained, the uncertainty in the value of $\alpha_3(M_Z)$ given in fig.s 3 is about $\Delta\alpha_3(M_Z) \approx \pm 2 \cdot 10^{-3}$.
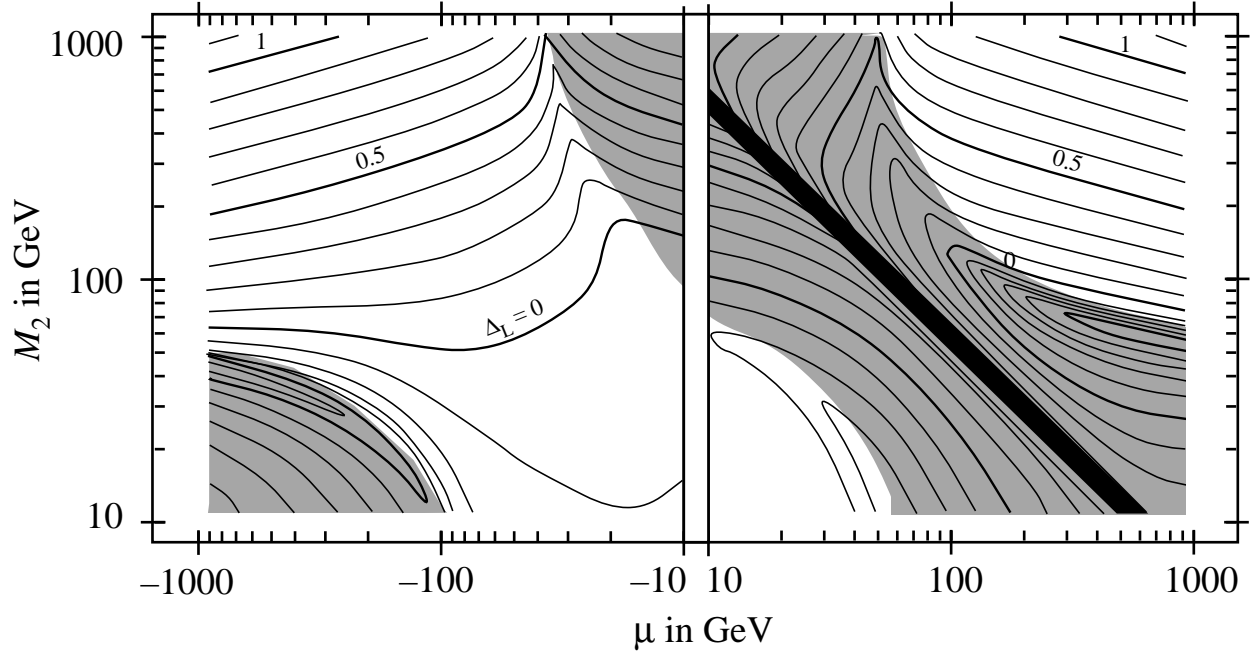
Fig. 2a: $\tan\beta = 1.5$

M_2 in GeV

μ in GeV
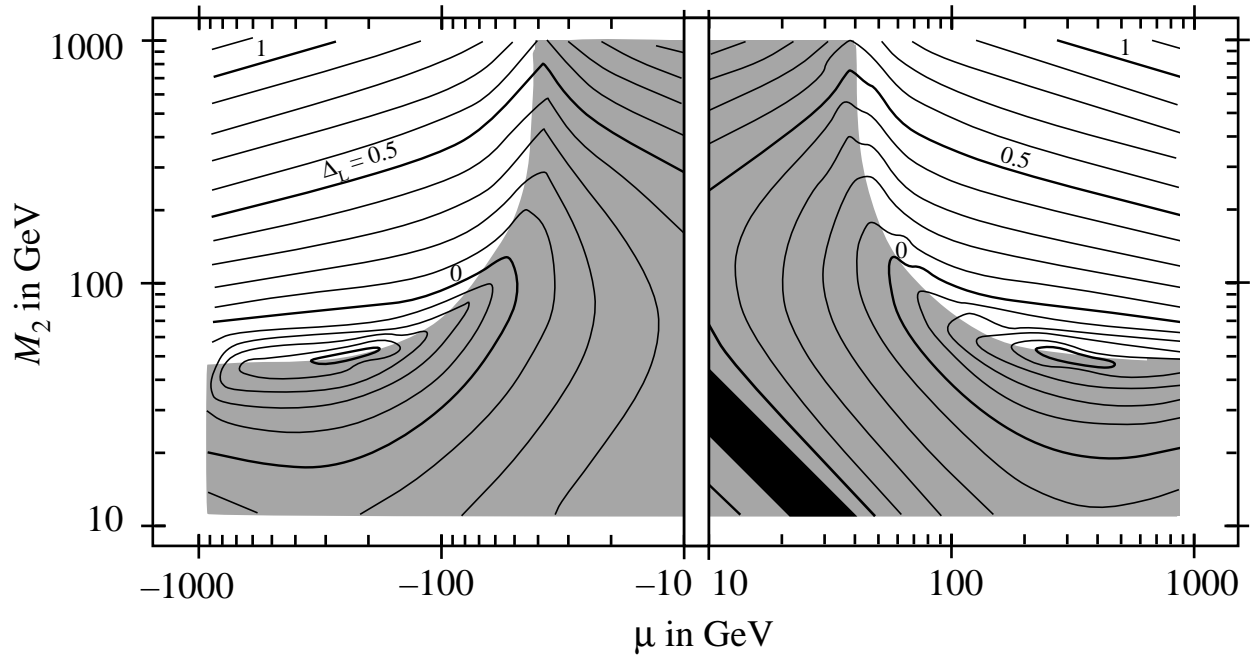

Fig. 2b: $\tan\beta = 40$

M_2 in GeV

μ in GeV

Figure 2: contribution to Δ_{L} from charginos and neutralinos for $\tan\beta = 1.5$ (fig. 2a) and $\tan\beta = 40$
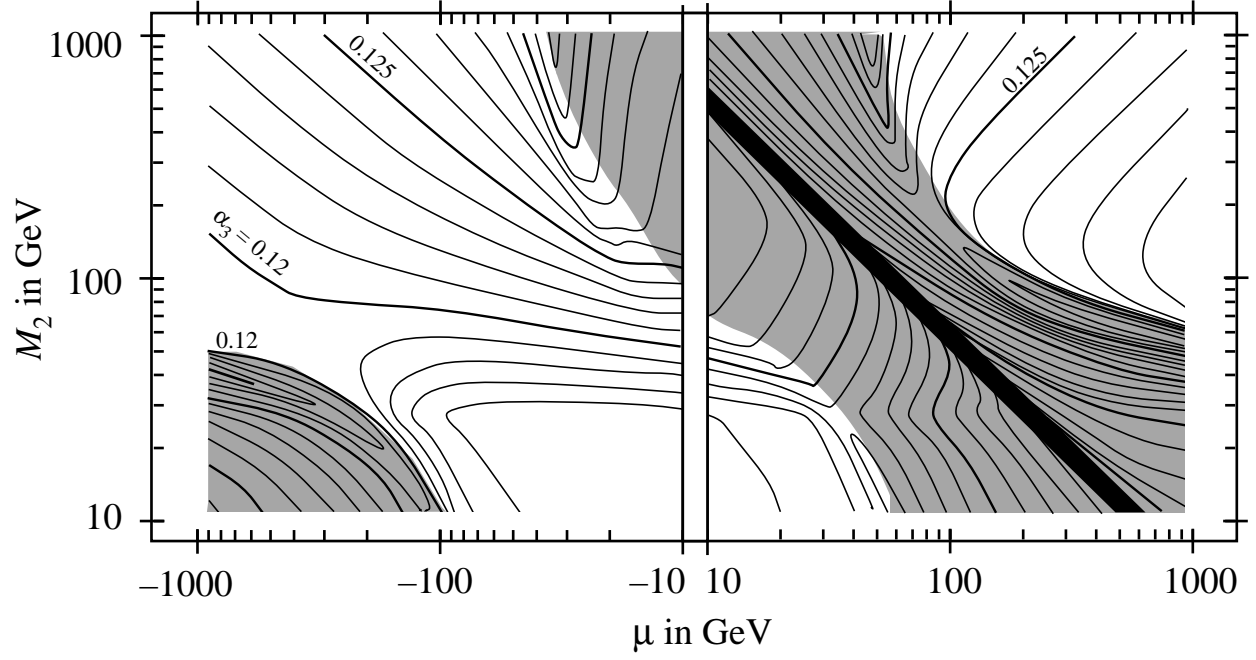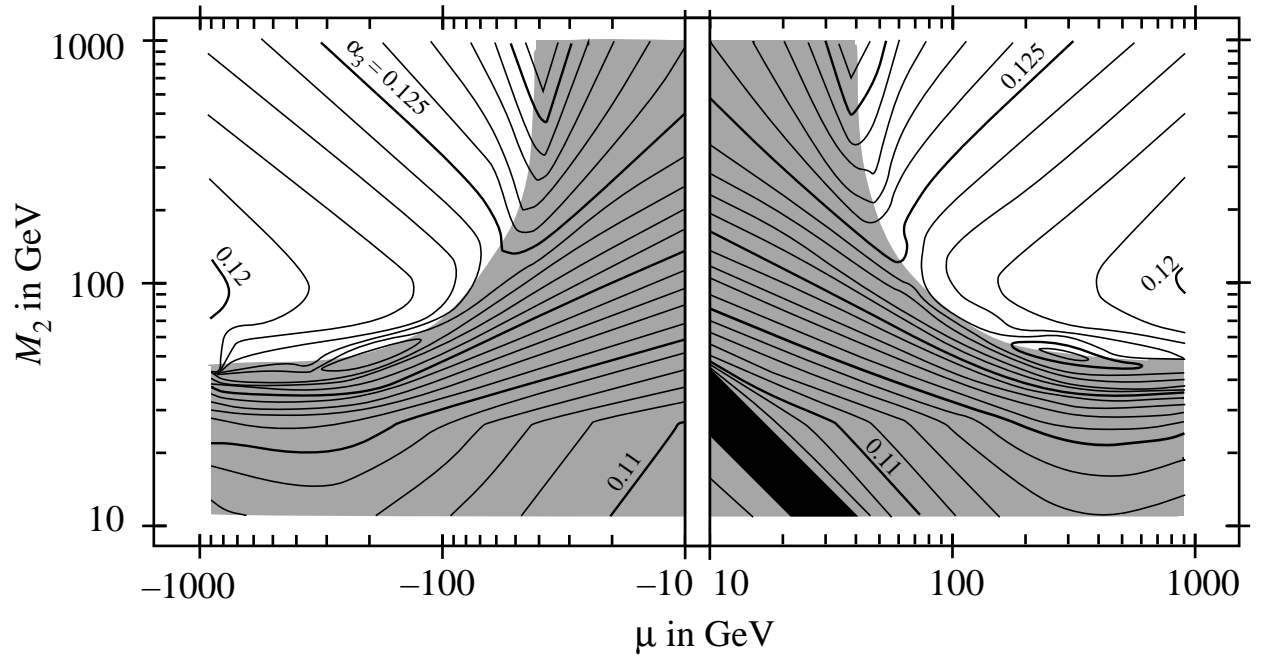
Figure 3: $\alpha_3(M_Z)$ in the plane (μ, M_2) for $\tan\beta = 1.5$ (fig. 3a) and $\tan\beta = 40$ (fig. 3b). Here the top pole

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